1.

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$$\arccos\frac{1}{3}+2\arccos\frac{1}{\sqrt{3}}=\pi$$

Proof. Let $a\equiv\arccos\frac{1}{3}$, $b\equiv\arccos\frac{1}{\sqrt{3}}$, then $\cos a=\frac{1}{3}$, $\sin a=\sqrt{1-\cos^2 a}=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3}$, $\cos b=\frac{1}{\sqrt{3}}$, $\sin b=\sqrt{1-\frac{1}{3}}=\frac{\sqrt{2}}{\sqrt{3}}$ and from the summation formulas for sinus and cosinus follows:

$\cos a+2b=\cos a\cos 2b-\sin a\sin 2b=\cos a\left(\cos^2 b-\sin^2 b\right)-\sin a\left(2\sin b\cos b\right)=$ $=\frac{1}{3}\left(\frac{1}{3}-\frac{2}{3}\right)-\frac{2\sqrt{2}}{3}\left(2\frac{1}{\sqrt{3}}\frac{\sqrt{2}}{\sqrt{3}}\right)=-\frac{1}{9}-\frac{8}{9}=-1$, hence $a+2b=(2k+1)\pi$ where $k\in\mathbb{Z}$. $k=0$ because both a and b are between 0 and π. □

Date: 5.11.2021.