JIŘÍ JANÁČEK

Let $0 \le k \le n$,

$$g(t) \equiv \sum_{j=0}^{n} a_j \frac{t^j}{j!}$$

and

$$f(x) \equiv 1 - e^{-x} \sum_{l=0}^{k-1} \frac{x^l}{l!},$$

then

$$(f \circ g)^{(i)}(0) = -\sum_{P \in B_I} \nabla^{\#P-1} p_{k-1}(a_0) \prod_{U \in P} a_{\#U}$$

 $\left(f\circ g\right)^{(i)}(0)=-\sum_{P\in B_I}\nabla^{\#P-1}p_{k-1}\left(a_0\right)\prod_{U\in P}a_{\#U}$ where $\left(f\circ g\right)(t)\equiv f\left(g\left(t\right)\right),\,\#S$ is cardinality of set $S,\,\#I=i,\,B_I$ is set of partitions of $I,\,p_j\left(x\right)\equiv\frac{x^j}{j!}e^{-x}$ if $j\geq 0$ and $p_j\equiv 0$ otherwise and ∇^j is j-th backward difference operator.

Proof. Faa di Bruno formula

Date : 5.12.2021.